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**Problem Description**

The first problem we encounter in the project is the conditional probability. This problem consists in two events A and B defined over some sample space. Consider a case that the two events are related in a way that, the probability of one of them depends on whether the other event has occurred, and for this reason it would be the term of conditional probability. The purpose is to simulate the rolling of two fair dice and obtain estimates of conditional probabilities under different scenarios using the computer to compute it. In the second problem of the project was the Base Rate Fallacy in which we use the notion of conditional probability to examine the so-called base rate fallacy. The base rate fallacy is an error that occurs when the conditional probability of some hypothesis H, given some evidence E, is assessed without taking sufficient account of the base rate or prior probability of H²

**Analysis of the Problem**

For this Project, the first problem is a conditional probability problem. First, we have to run the cond.m file and increase the number of rolls to get a more precise answer. Every execution of the program will display result. This result will be displayed on a table as problem requires. For the next steps we must modify function and change values of A and B to get the desired results from the program. The more attempts you have, the more accurate the probability and exact the result. The second problem is Base rate fallacy problem. For this problem run the baserate.m code a few times and insert the result in the Table 3 and compare with the exact probability. Problem two of the base rate fallacy is to detect whether or not a person is infected. We must modify the baserate.m code to have the exact value and the probability. Then we must insert these values on the table to compare the estimated and the exact.

**Solution Techniques**

**Problem I:** Conditional Probability

In problem one we are presented with two dice with a series of events given by the teacher. To solve these events, we use the following methods**:**

**Table 1:** Simulation of dice

|  |  |  |
| --- | --- | --- |
| **Dice A** | **Dice B** | **Total** |
| 1 | 1 | 2 |
| 1 | 2 | 3 |
| 1 | 3 | 4 |
| 1 | 4 | 5 |
| 1 | 5 | 6 |
| 1 | 6 | 7 |
| 2 | 1 | 3 |
| 2 | 2 | 4 |
| 2 | 3 | 5 |
| 2 | 4 | 6 |
| 2 | 5 | 7 |
| 2 | 6 | 8 |
| 3 | 1 | 4 |
| 3 | 2 | 5 |
| 3 | 3 | 6 |
| 3 | 4 | 7 |
| 3 | 5 | 8 |
| 3 | 6 | 9 |
| 4 | 1 | 5 |
| 4 | 2 | 6 |
| 4 | 3 | 7 |
| 4 | 4 | 8 |
| 4 | 5 | 9 |
| 4 | 6 | 10 |
| 5 | 1 | 6 |
| 5 | 2 | 7 |
| 5 | 3 | 8 |
| 5 | 4 | 9 |
| 5 | 5 | 10 |
| 5 | 6 | 11 |
| 6 | 1 | 7 |
| 6 | 2 | 8 |
| 6 | 3 | 9 |
| 6 | 4 | 10 |
| 6 | 5 | 11 |
| 6 | 6 | 12 |

* **Step 1:** Dice A : absolute value of difference of dice equals 2.

Dice B: sum of dice equals 8

Using Table 1:

|  |  |  |
| --- | --- | --- |
| **Dice A** | **Dice B** | **Total** |
| 1 | 1 | 2 |
| 1 | 2 | 3 |
| 1 | 3 | 4 |
| 1 | 4 | 5 |
| 1 | 5 | 6 |
| 1 | 6 | 7 |
| 2 | 1 | 3 |
| 2 | 2 | 4 |
| 2 | 3 | 5 |
| 2 | 4 | 6 |
| 2 | 5 | 7 |
| 2 | 6 | 8 |
| 3 | 1 | 4 |
| 3 | 2 | 5 |
| 3 | 3 | 6 |
| 3 | 4 | 7 |
| 3 | 5 | 8 |
| 3 | 6 | 9 |
| 4 | 1 | 5 |
| 4 | 2 | 6 |
| 4 | 3 | 7 |
| 4 | 4 | 8 |
| 4 | 5 | 9 |
| 4 | 6 | 10 |
| 5 | 1 | 6 |
| 5 | 2 | 7 |
| 5 | 3 | 8 |
| 5 | 4 | 9 |
| 5 | 5 | 10 |
| 5 | 6 | 11 |
| 6 | 1 | 7 |
| 6 | 2 | 8 |
| 6 | 3 | 9 |
| 6 | 4 | 10 |
| 6 | 5 | 11 |
| 6 | 6 | 12 |

**Legend:**

* Yellow: Event A
* Green: Event B
* Light Blue: Pr[A|B]

**Process:**

**Event A:** (1,3), (2,4), (3,5), (4,2), (4,6), (5,3), (6,4)

Pr[A] = 7/36 = 0.19%

**Event B:** (2,6), (3,5), (4,4), (5,3), (6,2)

Pr[B]= 5/36= 0.138%

Pr[A ∩ B]= 2/36

Pr[A|B] = 2/36 / 5/36 = 2/5 = 0.4%

* **Step 2:** Dice A: first die equals 3

Dice B: sum of dice is less than 8

Using Table 1:

|  |  |  |
| --- | --- | --- |
| **Dice A** | **Dice B** | **Total** |
| 1 | 1 | 2 |
| 1 | 2 | 3 |
| 1 | 3 | 4 |
| 1 | 4 | 5 |
| 1 | 5 | 6 |
| 1 | 6 | 7 |
| 2 | 1 | 3 |
| 2 | 2 | 4 |
| 2 | 3 | 5 |
| 2 | 4 | 6 |
| 2 | 5 | 7 |
| 2 | 6 | 8 |
| 3 | 1 | 4 |
| 3 | 2 | 5 |
| 3 | 3 | 6 |
| 3 | 4 | 7 |
| 3 | 5 | 8 |
| 3 | 6 | 9 |
| 4 | 1 | 5 |
| 4 | 2 | 6 |
| 4 | 3 | 7 |
| 4 | 4 | 8 |
| 4 | 5 | 9 |
| 4 | 6 | 10 |
| 5 | 1 | 6 |
| 5 | 2 | 7 |
| 5 | 3 | 8 |
| 5 | 4 | 9 |
| 5 | 5 | 10 |
| 5 | 6 | 11 |
| 6 | 1 | 7 |
| 6 | 2 | 8 |
| 6 | 3 | 9 |
| 6 | 4 | 10 |
| 6 | 5 | 11 |
| 6 | 6 | 12 |

**Process:**

**Event A:** (3,1), (3,2), (3,3), (3,4), (3,5), (3,6)

Pr[A] = 6/36 = 0.16%

**Event B:** (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (5,1), (5,2), (6,1)

Pr[B]= 21/36= 0.58%

Pr[A ∩ B]= 4/36

Pr[A|B] = 4/36 / 21/36 = 4/21 = 0.1904%

* **Step 3:** Dice A: Product of dice is at least 15

Dice B: second die does not equal 5

Using Table 1:

|  |  |  |
| --- | --- | --- |
| **Dice A** | **Dice B** | **Total** |
| 1 | 1 | 2 |
| 1 | 2 | 3 |
| 1 | 3 | 4 |
| 1 | 4 | 5 |
| 1 | 5 | 6 |
| 1 | 6 | 7 |
| 2 | 1 | 3 |
| 2 | 2 | 4 |
| 2 | 3 | 5 |
| 2 | 4 | 6 |
| 2 | 5 | 7 |
| 2 | 6 | 8 |
| 3 | 1 | 4 |
| 3 | 2 | 5 |
| 3 | 3 | 6 |
| 3 | 4 | 7 |
| 3 | 5 | 8 |
| 3 | 6 | 9 |
| 4 | 1 | 5 |
| 4 | 2 | 6 |
| 4 | 3 | 7 |
| 4 | 4 | 8 |
| 4 | 5 | 9 |
| 4 | 6 | 10 |
| 5 | 1 | 6 |
| 5 | 2 | 7 |
| 5 | 3 | 8 |
| 5 | 4 | 9 |
| 5 | 5 | 10 |
| 5 | 6 | 11 |
| 6 | 1 | 7 |
| 6 | 2 | 8 |
| 6 | 3 | 9 |
| 6 | 4 | 10 |
| 6 | 5 | 11 |
| 6 | 6 | 12 |

**Process:**

**Event A:** (3,5), (5,3)

Pr[A] = 2/36 = 1/18 = 0.05%

**Event B:** (1,1), (1,2), (1,3), (1,4), (1,6), (2,1), (2,2), (2,3), (2,4), (2,6), (3,1), (3,2), (3,3), (3,4), (3,6), (4,1), (4,2), (4,3), (4,4), (4,6), (5,1), (5,2), (5,3), (5,4), (5,6), (6,1), (6,2), (6,3), (6,4), (6,6)

Pr[B]= 30/36= 5/6 = 0.83%

Pr[A ∩ B]= 1/36

Pr[A|B] = 1/36 / 30/36 = 1/30 = 0.03%

* **Step 4:** Dice A: at least one die equals 4

Dice B: both dice are even

Using Table 1:

|  |  |  |
| --- | --- | --- |
| **Dice A** | **Dice B** | **Total** |
| 1 | 1 | 2 |
| 1 | 2 | 3 |
| 1 | 3 | 4 |
| 1 | 4 | 5 |
| 1 | 5 | 6 |
| 1 | 6 | 7 |
| 2 | 1 | 3 |
| 2 | 2 | 4 |
| 2 | 3 | 5 |
| 2 | 4 | 6 |
| 2 | 5 | 7 |
| 2 | 6 | 8 |
| 3 | 1 | 4 |
| 3 | 2 | 5 |
| 3 | 3 | 6 |
| 3 | 4 | 7 |
| 3 | 5 | 8 |
| 3 | 6 | 9 |
| 4 | 1 | 5 |
| 4 | 2 | 6 |
| 4 | 3 | 7 |
| 4 | 4 | 8 |
| 4 | 5 | 9 |
| 4 | 6 | 10 |
| 5 | 1 | 6 |
| 5 | 2 | 7 |
| 5 | 3 | 8 |
| 5 | 4 | 9 |
| 5 | 5 | 10 |
| 5 | 6 | 11 |
| 6 | 1 | 7 |
| 6 | 2 | 8 |
| 6 | 3 | 9 |
| 6 | 4 | 10 |
| 6 | 5 | 11 |
| 6 | 6 | 12 |

**Process:**

**Event A:** (4,1), (4,2), (4,3), (4,4), (4,5), (4,6)

Pr[A] = 6/36 = 1/6 = 0.16%

**Event B:** (1,1), (2,2), (3,3), (4,4), (5,5), (6,6)

Pr[B]= 6/36= 1/6 = 0.16%

Pr[A ∩ B]= 1/36

Pr[A|B] = 1/36 / 6/36 = 1/6 = 0.16%

**Problem II:** The Base Rate Fallacy

Next, we will show all the procedures to solve the problem two:

**Problem II: Experiment A**

Blood test:

If we define our hypothesis, H, and evidence, E, to be

H = {being infected},

E = {blood test comes back positive}

Pr[E|H] = 0.95, Pr[E|Hc ] = 0.01, and Pr[H] = 0.005 (base rate or prior probability)

Pr[H|E] = Pr[E|H] Pr[H] / Pr[E|H] Pr[H] + Pr[E|Hc ] Pr[Hc ]

Pr[H|E] = (0.95)(0.005) / (0.95)(0.005) + (0.01)(0.995) = 0.3231%

**Problem II: Experiment B**

Jimmy’s Urn:

H = {being draw to marble from the urn},

E = {evidence}

Pr[E|H] = 0.8, Pr[E|Hc ] = 0.5, and Pr[H] = 0.1 (base rate or prior probability)

Pr[H|E] = Pr[E|H] Pr[H] / Pr[E|H] Pr[H] + Pr[E|Hc ] Pr[Hc ]

Pr[H|E] = (0.8)(0.1) / (0.8)(0.1) + (0.5)(0.995) = 0.1385%

**MATLAB Programs**

1. **Conditional Probability**

Steps:

1. Download the MATLAB file cond.m from the course web page. The file contains a program that simulates two fair dice. The program defines two events: the event A that the absolute value of the difference of the dice equals 2; and the event B that the sum of the dice equals 8. Execute the program a few times. You may need to increase the number of rolls in order to get reasonable approximations to Pr[A|B]. Enter a representative simulation result in Table 1 at the end of this subsection

We ran this step 3 times increasing the number of rolls.

* Original code(number of rolls 10000):

% cond.m

% Experiment in conditional probability.

% Simulates the rolling of two fair dice.

clear all; close all; clc;

% Number of rolls

n = 10000;

% Points shown on each die

p1 = ceil(6\*rand(1,n));

p2 = ceil(6\*rand(1,n));

% Define event A = absolute value of difference of dice is 2

a = (abs(p1 - p2) == 2);

% Define event B = sum of dice is 8

b = ((p1 + p2) == 8);

% Conditional probability Pr[A|B]

pab = sum(a & b)/n; % Pr[AB]

pa = sum(a)/n; % Pr[A]

pb = sum(b)/n; % Pr[B]

pagb = pab/pb;

disp('Rolling of two fair dice:');

disp('A = absolute value of difference of dice is 2');

disp('B = sum of dice is 8');

disp(['Estimated probability Pr[A] : ',num2str(pa)]);

disp(['Estimated conditional probability Pr[A|B]: ',num2str(pagb)]);

* Modified code Step 1(number of rolls 20000):

% cond.m

% Experiment in conditional probability.

% Simulates the rolling of two fair dice.

clear all; close all; clc;

% Number of rolls

n = 20000;

% Points shown on each die

p1 = ceil(6\*rand(1,n));

p2 = ceil(6\*rand(1,n));

% Define event A = absolute value of difference of dice is 2

a = (abs(p1 - p2) == 2);

% Define event B = sum of dice is 8

b = ((p1 + p2) == 8);

% Conditional probability Pr[A|B]

pab = sum(a & b)/n; % Pr[AB]

pa = sum(a)/n; % Pr[A]

pb = sum(b)/n; % Pr[B]

pagb = pab/pb;

disp('Rolling of two fair dice:');

disp('A = absolute value of difference of dice is 2');

disp('B = sum of dice is 8');

disp(['Estimated probability Pr[A] : ',num2str(pa)]);

disp(['Estimated conditional probability Pr[A|B]: ',num2str(pagb)]);

* Modified code Step 1(number of rolls 30000):

% cond.m

% Experiment in conditional probability.

% Simulates the rolling of two fair dice.

clear all; close all; clc;

% Number of rolls

n = 30000;

% Points shown on each die

p1 = ceil(6\*rand(1,n));

p2 = ceil(6\*rand(1,n));

% Define event A = absolute value of difference of dice is 2

a = (abs(p1 - p2) == 2);

% Define event B = sum of dice is 8

b = ((p1 + p2) == 8);

% Conditional probability Pr[A|B]

pab = sum(a & b)/n; % Pr[AB]

pa = sum(a)/n; % Pr[A]

pb = sum(b)/n; % Pr[B]

pagb = pab/pb;

disp('Rolling of two fair dice:');

disp('A = absolute value of difference of dice is 2');

disp('B = sum of dice is 8');

disp(['Estimated probability Pr[A] : ',num2str(pa)]);

disp(['Estimated conditional probability Pr[A|B]: ',num2str(pagb)]);

1. Modify cond.m so that A is the event that the first die equals 3, and B is the event that the sum of the dice is less than 8. That is, you have to find logical expressions that encode these events and replace the appropriate lines of code in cond.m. Remember to save your modified program under a different file name. Then run your program and enter your simulation result in Table 1.

* Modified code:

% condm.m

% Experiment in conditional probability.

% Simulates the rolling of two fair dice.

clear all; close all; clc;

% Number of rolls

n = 10000;

% Points shown on each die

p1 = ceil(6\*rand(1,n));

p2 = ceil(6\*rand(1,n));

% Define event A = first dice equals 3

a = ((p1) == 3);

% Define event B = sum of dice is less than 8

b = ((p1 + p2) < 8);

% Conditional probability Pr[A|B]

pab = sum(a & b)/n; % Pr[AB]

pa = sum(a)/n; % Pr[A]

pb = sum(b)/n; % Pr[B]

pagb = pab/pb;

disp('Rolling of two fair dice:');

disp('A = first dice equals 3');

disp('B = sum of dice is less than 8');

disp(['Estimated probability Pr[A] : ',num2str(pa)]);

disp(['Estimated conditional probability Pr[A|B]: ',num2str(pagb)]);

1. Repeat the previous step for A, the event that the product of the dice is at least 15; and B, the event that the second die does not equal 5.

* Code

% condm1.m

% Experiment in conditional probability.

% Simulates the rolling of two fair dice.

clear all; close all; clc;

% Number of rolls

n = 10000;

% Points shown on each die

p1 = ceil(6\*rand(1,n));

p2 = ceil(6\*rand(1,n));

% Define event A = product of dice is at least 15

a = (p1 .\* p2)==15;

% Define event B = second dice does not equal 5

b = ((p2) ~= 5);

% Conditional probability Pr[A|B]

pab = sum(a & b)/n; % Pr[AB]

pa = sum(a)/n; % Pr[A]

pb = sum(b)/n; % Pr[B]

pagb = pab/pb;

disp('Rolling of two fair dice:');

disp('A = product of dice is at least 15');

disp('B = second dice does not equal 5');

disp(['Estimated probability Pr[A] : ',num2str(pa)]);

disp(['Estimated conditional probability Pr[A|B]: ',num2str(pagb)]);

1. Repeat Step 2 for A, the event that at least one die equals 4; and B, the event that both dice are even

* Code:

% condm2.m

% Experiment in conditional probability.

% Simulates the rolling of two fair dice.

clear all; close all; clc;

% Number of rolls

n = 10000;

% Points shown on each die

p1 = ceil(6\*rand(1,n));

p2 = ceil(6\*rand(1,n));

% Define event A = at least one dice equals 4

a = ((p1)== 4);

% Define event B = both dice are even

b = ((p2 == p1));

% Conditional probability Pr[A|B]

pab = sum(a & b)/n; % Pr[AB]

pa = sum(a)/n; % Pr[A]

pb = sum(b)/n; % Pr[B]

pagb = pab/pb;

disp('Rolling of two fair dice:');

disp('A = at least one dice equals 4');

disp('B = both dice are even');

disp(['Estimated probability Pr[A] : ',num2str(pa)]);

disp(['Estimated conditional probability Pr[A|B]: ',num2str(pagb)])

2. **The Base Rate Fallacy:**

Steps:

1. Download the MATLAB program baserate.m from the course web page. Please read the comments in the program to understand how it works. Run the program a few times and enter a representative result in Table 2 at the end of this subsection. Compare your simulation results with the exact probability Pr[H|E] = 0.3231, the probability of a person being infected given that the blood test is positive.

* Original Code

% baserate.m

% Program to test the base rate fallacy.

% Known information about a blood test:

% a) Pr[E|H] = 0.95, probability that test is positive

% if person is infected;

% b) Pr[E|H^c] = 0.01, probability that test is positive

% if person is NOT infected;

% c) Pr[H] = 0.005, probability of person being infected.

clear all; close all; clc;

% Number of trials

n = 100000;

% Event of test being positive for an infected person (E|H)

eh = (rand(1,n) <= 0.95);

% Event of test being positive for an uninfected person (E|H^c)

ehc = (rand(1,n) <= 0.01);

% Event of a person being infected person (H)

h = (rand(1,n) <= 0.005);

% Test is positive, regardless of whether the person is

% infected or not (E):

% 1) test is positive given that the person is infected (E|H)

% AND the person is infected (H);

% OR

% 2) test is positive given that the person is NOT infected (E|H^c)

% AND the person is NOT infected (H^c)

e = (eh & h) | (ehc & ~h);

% Conditional probability

% Pr[H|E] = Pr[E|H]\*Pr[H]/Pr[E] = Pr[EH]/Pr[E]

peh\_h = sum(eh & h)/n; % Pr[EH]

pe = sum(e)/n; % Pr[E]

phe = peh\_h/pe;

disp(' ');

disp(['Estimated conditional probability Pr[H|E] = ',num2str(phe)]);

disp(' ');

* Test with 800000 trials:

% Program to test the base rate fallacy.

% Known information about a blood test:

% a) Pr[E|H] = 0.95, probability that test is positive

% if person is infected;

% b) Pr[E|H^c] = 0.01, probability that test is positive

% if person is NOT infected;

% c) Pr[H] = 0.005, probability of person being infected.

clear all; close all; clc;

% Number of trials

n = 800000;

% Event of test being positive for an infected person (E|H)

eh = (rand(1,n) <= 0.95);

% Event of test being positive for an uninfected person (E|H^c)

ehc = (rand(1,n) <= 0.01);

% Event of a person being infected person (H)

h = (rand(1,n) <= 0.005);

% Test is positive, regardless of whether the person is

% infected or not (E):

% 1) test is positive given that the person is infected (E|H)

% AND the person is infected (H);

% OR

% 2) test is positive given that the person is NOT infected (E|H^c)

% AND the person is NOT infected (H^c)

e = (eh & h) | (ehc & ~h);

% Conditional probability

% Pr[H|E] = Pr[E|H]\*Pr[H]/Pr[E] = Pr[EH]/Pr[E]

peh\_h = sum(eh & h)/n; % Pr[EH]

pe = sum(e)/n; % Pr[E]

phe = peh\_h/pe;

disp(' ');

disp(['Estimated conditional probability Pr[H|E] = ',num2str(phe)]);

disp(' ');

1. Little Jimmy has an urn that contains 100 marbles. He tells you that he might have either 80 blue marbles and 20 red ones, or 50 blue and 50 red. Let these two hypotheses be H and Hc , respectively. Jimmy lets you randomly draw a marble from the urn. Suppose that you get a blue marble; this will be the evidence E. Do you think that Pr[H|E] is high simply because Pr[E|H] = 0.8 is higher than Pr[E|Hc ] = 0.5? Little Jimmy tells you a little secret. He tells you that the prior probability of H is very low; more specifically, Pr[H] = 0.1 – perhaps Jimmy has a preference for red marbles? With this secret revealed, do you think Pr[H|E] can be low as well? Given that you draw a blue marble, the probability that the urn had the 80 blue, 20 red mix might still be very low, while the probability it had the 50 blue, 50 red mix might yet be much higher. To ignore the implication of the low prior probability of H on the conditional probability Pr[H|E] is to commit the base rate fallacy.

* Code:

% baseratem.m

% Program to test the base rate fallacy.

% Little Jimmy Problem

%Pr[E|H] = 0.8

%Pr[E|H^c] = 0.5

% Pr[H] = 0.1

clear all; close all; clc;

% Number of trials

n = 100000;

%(E|H)

eh = (rand(1,n) <= 0.8);

% (E|H^c)

ehc = (rand(1,n) <= 0.5);

% (H)

h = (rand(1,n) <= 0.1);

e = (eh & h) | (ehc & ~h);

% Conditional probability

% Pr[H|E] = Pr[E|H]\*Pr[H]/Pr[E] = Pr[EH]/Pr[E]

peh\_h = sum(eh & h)/n; % Pr[EH]

pe = sum(e)/n; % Pr[E]

phe = peh\_h/pe;

disp(' ');

disp(['Estimated conditional probability Pr[H|E] = ',num2str(phe)]);

disp(' ');

**Tables and results**

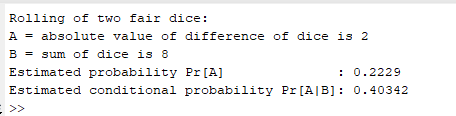
**Problem I:** Conditional Probability

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Events | | Estimates | | Exact | |
| A | B | Pr[A] | Pr[A|B] | Pr[A] | Pr[A|B] |
| absolute value of difference of dice equals 2 | sum of dice equals 8 | 0.2229% | 0.40342% | 0.19% | 0.4% |
| first dice equals 3 | sum of dice is less than 8 | 0.1689% | 0.19226% | 0.16% | 0.19% |
| product of dice is at least 15 | second die does not at equal 5 | 0.0535% | 0.030128% | 0.05% | 0.03% |
| at least one dice equals 4 | both dice are even | 0.1626% | 0.16064% | 0.16% | 0.16% |

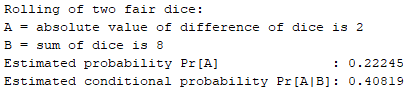
**Table 2:** Probabilities of events associated with two fair dice.

**Results**

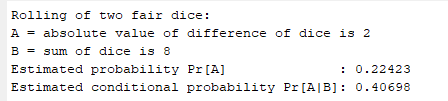
* Step 1 with 10000 rolls:



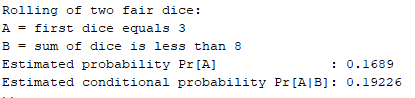
* Step 1 with 20000 rolls:



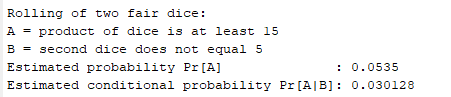
* Step 1 with 30000 rolls:



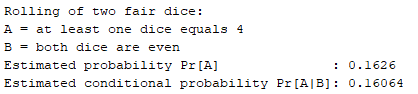
* Step 2:



* Step 3:



* Step 4:



**Problem II:** The Base Rate Fallacy

|  |  |  |
| --- | --- | --- |
| Experiment | Estimated Pr[H|E] | Exact Pr[H|E] |
| blood test | 0.3220% | 0.3231% |
| Jimmy’s urn | 0.15122% | 0.138% |

**Table 3:** Results for the base rate fallacy analysis

**Results**

* **Blood Test Experiment**



* **Jimmy’s urn Experiment**



**Questions**

1. Do you think that Pr[H|E] is high simply because Pr[E|H] = 0.8 is higher than Pr[E|Hc ] = 0.5?

Yes, because if we put Pr [E | H] less than Pr [E | Hc] the probability of Pr [H | E] will decrease. For example:

Pr[E|H] = 0.5, Pr[E|Hc ] = 0.8, and Pr[H] = 0.1 (base rate or prior probability)

Pr[H|E] = Pr[E|H] Pr[H] / Pr[E|H] Pr[H] + Pr[E|Hc ] Pr[Hc ]

Pr[H|E] = (0.5) (0.1) / (0.5) (0.1) + (0.8) (0.995) = 0.05910%

1. Little Jimmy tells you a little secret. He tells you that the prior probability of H is very low; more specifically, Pr[H] = 0.1 – perhaps Jimmy has a preference for red marbles?

No, Jimmy does not have any color preference since the probability Pr [Hc] being higher than the probability Pr [H] in his hypothesis suggests that the Hc has 50 blue and 50 red while in the H has 80 blue and 20 reds for this reason Jimmy has no preference for red.

1. With this secret revealed, do you think Pr[H|E] can be low as well?

No, because as long as Pr [E | H] is greater than Pr [E | Hc] the probability of Pr [H | E] will not decrease.

**References**

* Fischer, I. (2016). Classical Probability Distributions. Retrieved April 22, 2019.
* Matlab Commands. (n.d.). Retrieved April 22, 2019, from https://www.mathworks.com/help/matlab/entering-commands.html